THERMODYNAMIC OPTIMIZATION OF FIN GEOMETRY

By

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CERTIFICATE

This is to certify that the work on "Thermodynamic Optimization of Fin Geometry," by Shambhu Nath Shukla, has been carried out under my supervision and has not been submitted elsewhere for a degree.

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Shambhu Nath Shukla

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NOMENCLATURE

B	Frictional irreversibility parameter, equation (2.2.10)
B_1	Frictional irreversibility parameter, equation (2.4.23).
C_D	Drag coefficient.
C_f	Skin friction coefficient.
D	Pin fin diameter.
F_D	Drag force.
h	Heat transfer coefficient.
k_f	Thermal conductivity of fluid
k_s	Thermal conductivity of fin material.
L	Fin length.
m	Pin fin conduction parameter, equation (2.2.2).
m'	Rectangular fin conduction parameter, equation (2.3.2).
m''	Triangular fin conduction parameter, equation (2.4.6).
M	Parameter, equation (2.2.11).
M'	Parameter, equation (2.4.21).
N_s	Entropy generation number.
Nu	Nusselt number.
P	Property group, $(k_s/k_f)^{1/2}/Pr^{1/6}$.
Pr	Prandtl number.
q_B	Base heat transfer rate.
q_∞	Heat transfer from control volume to remaining fluid.
Re_D	Reynolds number, $U_\infty D/\nu$.
Re_L	Reynolds number, $U_\infty L/\nu$.

Re_W	Reynolds number, $U_\infty W/$.
Re_δ	Reynolds number, $U_\infty \delta/\nu$.
$Re_{\delta o}$	Reynolds number, $U_\infty \delta_o/\nu$.
S_{gen}	Entropy generation rate.
T_B	Absolute temperature of fin base.
T_o	Absolute temperature of environment.
T_∞	Absolute temperature of free stream.
u	Parameter, equation (2.4.13).
U_∞	Free stream velocity.
v	Parameter, equation (2.3.21).
W	Width of fin.
W_{lost}	Rate of lost available work.
x	Longitudinal coordinate.
y	Parameter, equation (2.3.14).
α	Half angle, Fig. 4.
γ	Slenderness ratio.
δ	Thickness of rectangular fin.
δ_o	Base thickness of triangular fin.
θ_B	Base-stream temperature difference, $T_B - T_\infty$.
ν	Kinematic viscosity of fluid.
ρ	Density of fluid.

ABSTRACT

A theoretical framework for the minimum entropy generation in extended surfaces or fins has been developed. The entropy generation equation comprises two terms, one due to heat transfer and the other due to fluid friction. The optimum geometry for a fin is obtained when the above two terms add to yield a minimum. Analytical solutions for three types of fins viz., a pin fin, a rectangular fin and a fin with a trangular profile are presented herein and the results plotted on graphs to determine their optimum dimensions. The thermodynamic optimization has been carried out for a given heat transfer rate and flow velocity.

CHAPTER I

INTRODUCTION

1.1 INTRODUCTION:

In the design of a system involving the generation or use of energy, the exergy method will provide the information to better select the component designs and their operating procedures that will be most effective for energy conservation. The minimization of lost work will produce the most efficient system. The exergy method of analysis is based on this principle. In this method, the primary task is to determine where the losses in the available work occur in the system, what the magnitude of these losses is, and which of the losses, when corrected, can most effectively improve the system efficiency. The selection of the losses in the system to be corrected should usually be based on economic considerations. However, the use of the available-work principle in the exergy method will bring about a significant improvement in the data used for making judgements, since it takes into account both the quantity and the quality of the inefficient energy utilization [1].

1.1.1 Exergy Losses and Irreversibility:

The exergy losses are produced in a system by the irreversible production of entropy caused by the non-ideal performance inherent in all real systems and components. Minimization of these irreversibilities is necessary in those places where the exergy loss to the system is quite significant. This aspect of the exergy-analysis method requires a knowledge of the degree of irreversibility and the associated exergy losses with mass and heat leakage, fluid-flow turbulence and friction. The reason for considering all exergy losses in a system during initial stages is that some processes that usually have a low exergy loss may show upto be the cause of a significant loss in another type of system. All irreversibilities in processes and systems should be given due consideration if effective energy-conservation measures are to be adopted.

1.1.2 Exergy Loss in Heat Exchangers:

Heat exchangers serve as components in a wide range of power and refrigeration equipment. In order to conserve available work, it is necessary to approach the design of heat exchangers from the point of view of entropy minimization. It is important to go further and focus attention on the components of heat exchangers and consider the design of each such building block for minimum irreversibility. Most of the heat exchangers used to-day are generally inefficient from an exergy stand point

because they are primarily designed on the basis of low cost that dictates a minimum-sized unit. To achieve this small sized heat exchanger goal, the temperature difference between the fluid streams is maximized. However, the larger the temperature difference in a heat exchanger, the greater is the exergy loss during heat transfer. Hence from exergy-saving point of view such designs are not desirable.

1.1.3 Loss to Surroundings:

The loss of thermal energy from a system to the surrounding environment is a direct loss of the exergy in the system. This loss is considered in the exergy method of system analysis as an **internal** loss and is, therefore, included in system efficiency calculations.

The first and second laws of thermodynamics, taken together, state that the entropy generated by any engineering system is proportional to the work lost irreversibly by the system [1]. This is expressed concisely as

$$W_{lost} = T_o \Sigma S_{gen} \quad (1.1.3.1)$$

where, W_{lost} is the lost available work or unavailability,

T_o is the absolute temperature of the environment, and

S_{gen} is the entropy generated in each component of the system.

Equation (1.1.3.1) implies that the thermodynamic irreversibility (entropy generation) of each system component contributes to the aggregate loss of available work in the system (W_{lost}). Therefore, it is important to concentrate on each component of the system and, by design, try to minimize the irreversibility (S_{gen}) of that component.

1.2 LITERATURE REVIEW:

The traditional approach to the optimization of fins consists of minimizing the consumption of fin material for the execution of a specified task. Duffin [2], on the basis of variational calculus proved that a two-dimensional fin must have a parabolic profile if it is to require the least material (volume) for a certain heat transfer rate. This design principle has been brought closer to reality by Razelos and Imre [3] who analysed the role of radiation, two-dimensional heat transfer, temperature dependent thermal conductivity and variable heat transfer coefficient. Many of these contributions have been summarised by Kern and Kraus [4]. The essence and practical limitations of this design philosophy have been discussed by Kraus and Snider [5].

An entirely different approach to the design of heat exchangers is based on the principle of minimization of exergy loss by the system sub-components. This approach was adopted

by A. Bejan [6] in the design of counter flow heat-exchangers for gas to gas application. His design is based on the thermodynamic irreversibility rate or useful power no longer available as a result of heat exchanger frictional pressure drop and stream to stream temperature differences. The irreversibility concept is used to establish a direct relationship between the heat exchanger design parameters and the useful power wasted due to heat exchanger non-ideality. Bejan's paper presents a heat exchanger design for a fixed irreversibility. In contrast to the traditional design procedures, the amount of the heat transferred between the streams and the pumping power for each side become the outputs of this design approach. D. Poulikakos and A. Bejan [10] found the fin geometry for minimum entropy generation in forced convection. They derived a general expression for the entropy generation rate for an arbitrary fin and minimizing this, obtained fin geometric parameters for various shapes corresponding to the minimum entropy generation rate. In their analysis, they assumed fin-base stream temperature difference θ_B to be very small compared to the absolute value of the free stream temperature. This assumption simplifies the expression for the entropy generation rate. In the present study, however, this assumption has been relaxed and an exact expression for the entropy generation rate has been used to optimize the fin geometry.

1.3 SCOPE OF PRESENT WORK:

This thesis outlines the procedure for the design of fins for minimum entropy generation in forced convection heat transfer. This design philosophy allows to properly account for the fact that fins, in addition to enhancing the heat transfer, increase the fluid friction also. The irreversibility minimization philosophy places this trade-off between heat transfer and fluid friction on solid foundations as heat transfer and fluid friction drag are both mechanisms for entropy generation. The competition between these two mechanisms, i.e., enhanced thermal contact and fluid friction is settled when the heat transfer irreversibility and the fluid friction irreversibility add to yield a minimum rate of entropy generation for the fin.

The thesis presents a derivation of the formula for the rate of the entropy generation for an arbitrary fin engaged in forced convection heat transfer. Based on the results **therefrom** it is shown how the geometric parameters of common fin shapes can be selected so that the fin saves most of the exergy while performing its specified heat transfer function. Throughout this study, the classical fin heat transfer model [4] has been adopted whereby the fin is slender enough so that the conduction process can be regarded as unidirectional. It is

further assumed that the properties of the fin material and those of the external fluid are constant. The external flow is assumed uniform and parallel to the base of the fin.

CHAPTER II

MATHEMATICAL FORMULATION

2.1 ENTROPY GENERATION DUE TO CONVECTIVE HEAT TRANSFER FROM A SINGLE FIN:

The entropy generated by a single fin in the cross flow can be evaluated based on the general model presented in Fig. 1. Considering, an arbitrary fin suspended in a uniform stream with the velocity, U_∞, and temperature, T_∞, the heat transfer-rate, q_B, is caused by the temperature difference between the fin base, T_B, and the free stream, T_∞. In addition, the cross-flow arrangement is responsible for a net drag force, F_D, which is transmitted through the fin to the base wall.

As shown in Fig. 1, we choose a control volume which is fixed relative to the fluid. The environment (the wall) drags the fin with a speed, U_∞, through the stationary fluid, by applying a force F_D tangentially to the control surface. In the steady state, the first and second laws of thermodynamics dictate, respectively,

$$q_B - q_\infty + F_D U_\infty = 0 \tag{2.1.1}$$

$$S_{gen} = \frac{q_\infty}{T_\infty} - \frac{q_B}{T_B} > 0 \tag{2.1.2}$$

where q_∞ is the net heat transfer rate from the control volume to the rest of the fluid, and S_{gen} is the rate of entropy generation associated with the heat and fluid flow arrangement. q_B is the heat flow rate at the base of fin and $F_D U_\infty$ represents heat dissipation due to friction.

Combining equations (2.1.1) and (2.1.2) yields

$$S_{gen} = \frac{q_B + F_D U_\infty}{T_\infty} - \frac{q_B}{T_B}$$

$$= q_B \left(\frac{1}{T_\infty} - \frac{1}{T_B} \right) + \frac{F_D U_\infty}{T_\infty}$$

$$= \frac{q_B \theta_B}{T_\infty T_B} + \frac{F_D U_\infty}{T_\infty} \qquad (2.1.3)$$

where $\theta_B = (T_B - T_\infty)$ is the temperature difference between the base of the fin and the free stream.

Equation (2.1.3) can be rewritten as

$$S_{gen} = \frac{q_B \theta_B}{T_\infty^2 (1 + \theta_B / T_\infty)} + \frac{F_D U_\infty}{T_\infty} \qquad (2.1.4)$$

The entropy generation rate, given by equation (2.1.4) demonstrates that inadequate thermal conductance and fluid friction contribute hand-in-hand to the degradation of the fin thermodynamic performance. The first term on the right hand side of the equation represents the entropy generation due to heat transfer across a non-zero temperature difference,

while the second term is the entropy generated due to fluid friction,

$$S_{gen} = (S_{gen})_{heat\ transfer} + (S_{gen})_{fluid\ friction} \qquad (2.1.5)$$

In order to minimize the heat transfer contribution to S_{gen}, one must minimize the base stream temperature difference θ_B. In practical terms, however, to minimise θ_B would imply the use of an infinitely large fin for a constant heat flow rate, such fin would be impractical and thermodynamically undesirable because an infinitely large fin would have an infinite (S_{gen}) fluid friction, hence an infinite total S_{gen}. Thus, although, fin size influences monotonically the base stream temperature difference, it plays a crucial trade-off role in the thermodynamic performance of the fin.

In this thesis, the fin size trade-off by minimizing the rate of entropy generation has been examined in detail for some of the most common fin geometries encountered in practice.

The size trade-offs presented later are based on minimizing equation (2.1.4) subject to following constraints:

1. Constant q_B, T_∞, U_∞ ,
2. Constant fluid properties,
3. Constant fin material properties.

2.2 PIN FIN

Pin fin geometry depends on only two dimensions: the length, L, and the diameter of the circular cross-section, D. According to unidirectional heat conduction model described in the introduction the relationship between the base heat flow rate and the base stream temperature difference is [4]

$$\theta_B = \frac{q_B}{\frac{\pi}{4} k_s D^2 m \tanh(mL)} \tag{2.2.1}$$

where,

$$m = (4h/k_s D)^{1/2} \tag{2.2.2}$$

and h and k_s are the heat transfer coefficient and the thermal conductivity of the fin material, respectively.

Substituting this expression for θ_B into equation (2.1.4) gives the total entropy generation rate as

$$S_{gen} = q_B^2/\left[\frac{\pi}{4} T_\infty^2 k_s D^2 m \tanh(mL)\left(1 + \frac{q_B}{\frac{\pi}{4} T_\infty k_s D^2 m \tanh(mL)}\right)\right] + \frac{F_D U_\infty}{T_\infty}$$

This can be simplified to

$$S_{gen} = q_B^2/\left[\frac{\pi}{4} T_\infty^2 k_s D^2 m \tanh(mL) + q_B T_\infty\right] + F_D U_\infty/T_\infty \tag{2.2.3}$$

Heat transfer coefficient h can be expressed as

$$h = (Nu\ k_f/D) = (Nu\ K_f\ U_\infty/Re_D\,\nu) \qquad (2.2.4)$$

where is the kinematic viscosity of the fluid, k_f is the thermal conductivity of the fluid and Re_D is the Reynolds number based on the diameter of the pin fin.

Now equation (2.2.2) can be simplified to

$$m = (4h/k_sD)^{1/2} = ((4k_f\ U_\infty\ Nu\ U_\infty)/(Re_D\,\nu.k_s\ Re_D\,\nu))^{1/2}$$

Thus $m = 2(k_f/k_s)^{1/2}\ Nu^{1/2}.\ (U_\infty/(Re_D\,\nu))$ (2.2.5)

and $L = (Re_L\,\nu\ /U_\infty)$ (2.2.6)

Re_L is the Reynolds number based on length of the fin.

Force F_D on the fin is given by.

$$F_D = C_D.(\ \tfrac{1}{2}\rho\,U_\infty^{\ 2}\ D.L) \qquad (2.2.7)$$

where C_D is the drag coefficient.

Combining equations (2.2.3) to (2.2.7), we get the following equation for the entropy generation rate

$$S_{gen} = q_B^{\ 2}/(\tfrac{\pi}{2}\ T_\infty^{\ 2}(k_fk_s)^{1/2}Nu^{1/2}\ Re_D\ \frac{\nu}{U_\infty}\ \tanh(2Nu^{1/2}$$

$$(k_f/k_s)^{1/2}\ Re_L/Re_D) + q_BT_\infty) + \rho\,\nu^2U_\infty\ Re_L\ Re_DC_D/2T_\infty$$

(2.2.8)

This equation is non-dimensionalised to yield the entropy generation number [9] as

$$N_s = S_{gen}/(q_B{}^2 U_\infty/k_s \nu T_\infty{}^2)$$

$$= (\frac{\pi}{2}(k_f/k_s)^{1/2} Nu^{1/2} Re_D \tanh[2Nu^{1/2}(k_f/k_s)^{1/2} Re_L/Re_D] + M)^{-1} + \frac{1}{2} BC_D Re_L Re_D \qquad (2.2.9)$$

where B is a fixed dimensionless parameter that accounts for the importance of the fluid friction irreversibility relative to the heat transfer irreversibility,

$$B = \rho \nu^3 k_s T_\infty / q_B{}^2 \qquad (2.2.10)$$

M is also a dimensionless parameter given by

$$M = (q_B U_\infty / k_s \nu T_\infty) \qquad (2.2.11)$$

Parameters B and M are known as soon as fluid properties, flow velocity, its temperature and base heat flow rate are specified.

If the fin is slender, the Nusselt number and the drag coefficient can be evaluated from the following results developed for a single cylinder in cross flow [11]

$$4 < Re_D < 40, \quad Nu = 0.919\, Re_D{}^{0.385}\, Pr^{1/3}$$

$$C_D = 5.484\, Re_D{}^{-0.246}$$

$$40 < Re_D < 4 \times 10^3, \quad Nu = 0.683\, Re_D^{0.466}\, Pr^{1/3}$$

$$C_D = 5.484\, Re_D^{-0.246}$$

$$4 \times 10^3 < Re_D < 4 \times 10^4, \quad Nu = 0.195\, Re_D^{0.618}\, Pr^{1/3}$$

$$C_D = 1.1$$

$$4 \times 10^4 < Re_D < 2 \times 10^5, \quad Nu = 0.0268\, Re_D^{0.805}\, Pr^{1/3}$$

$$C_D = 1.1 \qquad (2.2.12)$$

The entropy generation number, N_s is a function of six dimensionless groups, two pertaining to the fin geometry (Re_L, Re_D), and four accounting for the working fluid and the fin-stream convective arrangement (Pr, k_s/k_f, B, M).

Minimization of N_s with respect to Re_L is achieved by solving

$$\partial N_s / \partial Re_L = 0.$$

Partial differentiation of equation (2.2.9) with respect to Re_L set equal to zero, yields

$$\frac{\partial N_s}{\partial Re_L} = -\pi\left(\frac{k_f}{k_s}\right) Nu\, \mathrm{Sech}^2 \left[2Nu^{1/2} \left(\frac{k_f}{k_s}\right)^{1/2} \frac{Re_L}{Re_D}\right]$$

$$+ \frac{1}{2} BC_D\, Re_D\left[\frac{\pi}{2} \left(\frac{k_f}{k_s}\right)^{1/2} Nu^{1/2}\, Re_D \tanh \left[2Nu^{1/2}\right.\right.$$

$$\left.\left.\left(\frac{k_f}{k_s}\right)^{1/2} \frac{Re_L}{Re_D}\right] + M\right]^2 = 0$$

or $$A' \tanh^2 (mL) + B' \tanh (mL) + C' = 0 \qquad (2.2.13)$$

where,

$$A' = \pi \frac{k_f}{k_s} Nu(1 + \frac{\pi}{8} B\ C_D\ Re_D^{\ 3})$$

$$B' = \frac{\pi}{2} B\ C_D\ Re_D^{\ 2}\ M\ (\frac{k_f}{k_s})^{1/2}\ Nu^{1/2}$$

$$C' = \frac{1}{2} B\ C_D\ Re_D\ M^2 - \pi\ (\frac{k_f}{k_s})\ Nu$$

$$mL = 2Nu^{1/2}\ (\frac{k_f}{k_s})^{1/2}\ \frac{Re_L}{Re_D}$$

Positive solution of equation (2.2.13) is given by

$$\tanh\ (mL) = \frac{-B' + (B'^2 - 4\ A'C')^{1/2}}{2A'} \tag{2.2.14}$$

Substituting expressions for A', B' and C' in (2.2.14) yields

$$\tanh\ (mL) = (-\ \frac{\sqrt{\pi}}{2} B\ C_D\ Re_D^{\ 2}\ M + [4\pi \frac{k_f}{k_s} Nu(1 + \frac{\pi}{8} B\ C_D\ Re_D^{\ 3}) - \frac{1}{2} B\ C_D\ Re_D\ M^2]^{1/2})/(2(\frac{\pi k_f Nu}{k_s})^{1/2}(1+ \frac{\pi}{8} B\ C_D\ Re_D^{\ 3})) \tag{2.2.15}$$

From equation (2.2.15), expression for Re_L,opt can be obtained as follows

$$mL = \tanh^{-1} [(-\ \frac{\sqrt{\pi}}{2} B\ C_D\ Re_D^{\ 2}M + (4\pi \frac{k_f}{k_s} Nu\ (1 + \frac{\pi}{8} B\ C_D\ Re_D^{\ 3}) - \frac{1}{2} B\ C_D\ Re_D\ M^2)^{1/2})/(2(\pi \frac{k_f}{k_s} Nu)^{1/2}(1+ \frac{\pi}{8} B\ C_D\ Re_D^{\ 3}))]$$

or,

$$Re_{L,opt} = \frac{Re_D}{2}\left(\frac{k_s}{k_f Nu}\right)^{1/2} \tanh^{-1} [(-\frac{\sqrt{\pi}}{2} B\ C_D\ Re_D^{\ 2} M +$$

$$[\ 4\pi \frac{k_f}{k_s} Nu\ (1 + \frac{\pi}{8} B\ C_D\ Re_D^{\ 3}) - \frac{1}{2} B\ C_D\ Re_D$$

$$M^2]^{1/2})/(2(\pi \frac{k_f}{k_s} Nu)^{1/2}\ (1 + \frac{\pi}{8} B\ C_D\ Re_D^{\ 3}))] \qquad (2.2.16)$$

The engineering significance of equation (2.2.16) is that the optimum fin length can be calculated immediately, provided Re_D is specified. Substituting equation (2.2.16) into equation (2.2.9), the entropy generation number N_s, corresponding to optimum pin length is obtained. This new function is dependent only on the diameter of the fin as far as the geometric parameters are concerned, and it is minimized numerically as shown in Fig. 5. The expression for N_s for optimum pin length is given as follows

$$N_s = [\frac{\pi}{2} Nu^{1/2}\ (\frac{k_f}{k_s})^{1/2}\ Re_D\ [(-\frac{\sqrt{\pi}}{2} B\ C_D\ Re_D^{\ 2}\ M +$$

$$[4\pi \frac{k_f}{k_s} Nu\ (1+ \frac{\pi}{8} B\ C_D\ Re_D^{\ 3}) - \frac{1}{2} B\ C_D\ Re_D\ M^2]^{1/2})/(2$$

$$(\pi \frac{k_f}{k_s} Nu)^{1/2}(1 + \frac{\pi}{8} B\ C_D\ Re_D^{\ 3}))] + M]^{-1} + \frac{1}{4}$$

$$\frac{B\ C_D\ Re_D^{\ 2}}{(Nu)^{1/2}}\ (\frac{k_s}{k_f})^{1/2} \tanh^{-1} [(-\frac{\sqrt{\pi}}{2} B\ C_D\ Re_D^{\ 2} M +$$

$$[4\pi \frac{k_f}{k_s} Nu(1 + \frac{\pi}{8} B\ C_D\ Re_D^{\ 3}) - \frac{1}{2} B\ C_D\ Re_D\ M^2])^{1/2})/(2$$

$$(\pi \frac{k_f}{k_s} Nu)^{1/2}\ (1+ \frac{\pi}{8} B\ C_D\ Re_D^{\ 3}))] \qquad (2.2.17)$$

Equation (2.2.17) is numerically minimized to give $Re_{D,opt}$ which is substituted back into equation (2.2.16) to get $Re_{L,opt}$.

2.3 RECTANGULAR FIN:

This section focuses attention on fin geometries modelled as thin conducting plates parallel to the flow direction. The fin geometry analysed here is shown in Fig. 3. The minimum irreversibility design criterion of this fin requires the selection of three geometric parameters, viz., the length L, the breadth (length swept by fluid), W, and the plate thickness δ. It is assumed that the length and the breadth of the fin are large compared to its thickness, δ. The relationship between the heat transfer rate and the base fluid temperature difference is given as [4]

$$\theta_B = \frac{q_B}{k_s \; W \; m' \tanh(m'L)} \tag{2.3.1}$$

where $$m' = \left(\frac{2h}{k_s \delta}\right)^{1/2} \tag{2.3.2}$$

and h is the heat transfer coefficient

$$m'L = \left[\frac{2Nu \, (k_f/k_s)}{Re_\delta \, Re_W}\right]^{1/2} Re_L \tag{2.3.3}$$

where

$$Nu = \frac{hW}{k_f} ; \quad Re_\delta = \frac{U_\infty \delta}{\nu} ; \quad Re_W = \frac{U_\infty W}{\nu} \quad \text{and} \quad Re_L = \frac{U_\infty L}{\nu}$$

The laminar heat transfer and the skin friction results for two-dimensional flat plates as given in [12] are

$$Nu = \frac{hW}{k_f} = 0.664 \; Re_W^{1/2} \; Pr^{1/3} \quad (2.3.4)$$

$$C_f = \frac{F_D}{\rho U_\infty^2 \; WL} = 1.328 \; Re_W^{-1/2} \quad (2.3.5)$$

Using equations (2.1.4) and (2.3.1) the entropy generation rate for a rectangular fin in the laminar flow region becomes

$$S_{gen} = q_B^2/(T_\infty^2 k_s \delta \, W \, m' \tanh (m'L) + q_B \, T_\infty) + F_D U_\infty / T_\infty \quad (2.3.6)$$

The entropy generation number is given by

$$\begin{aligned} N_s &= S_{gen}/(q_B^2 U_\infty / k_s \nu T_\infty^2) \\ &= [(2Nu \frac{k_f}{k_s} Re_\delta \, Re_W)^{1/2} \tanh (m'L) + M]^{-1} \\ &\quad + BC_f \, Re_W \, Re_L \end{aligned} \quad (2.3.7)$$

Combination of equations (2.3.3) and (2.3.4) gives

$$m'L = 1.15 \, (\frac{k_f}{k_s})^{1/2} \, Pr^{1/2} \, Re_L \, Re_W^{-1/4} \, Re_\delta^{-1/2} \quad (2.3.8)$$

It is seen that the thickness, δ, appears only in the heat transfer term of the N_s equation (2.3.7), consequently, Re_δ does not play any role in the minimization process of N_s. Also, since in most practical cases, δ is determined by

consideration such as price, availability and machinability of sheet metal, it makes engineering sense to regard Re_δ as fixed. The minimization of N_s with respect to Re_W and Re_L is achieved by solving the following simultaneous set of equations

$$\frac{\partial N_s}{\partial Re_W} = 0 \qquad \text{and} \qquad \frac{\partial N_s}{\partial Re_L} = 0$$

Differentiating (2.3.7) with respect to Re_L and setting it equal to zero, yields

$$A'' \tanh^2 (m'L) + B'' \tanh (m'L) + C'' = 0 \qquad (2.3.9)$$

where,

$$A'' = 2Nu \frac{k_f}{k_s} (1 + B\, C_f\, Re_\delta\, Re_W^2)$$

$$B'' = 2\; 2\; B\, C_f\, M \left(Nu \frac{k_f}{k_s} Re_\delta\right)^{1/2} Re_W^{3/2}$$

and

$$C'' = B\, C_f\, Re_W\, M^2 - 2 \frac{k_f}{k_s} Nu$$

Solution of the above equation (2.3.9) is given by

$$\tanh (m'L) = \frac{-B'' + (B''^2 - 4A''C'')^{1/2}}{2A''} \qquad (2.3.10)$$

Substituting expressions for A'', B'' and C'' we get the following expression

$$\tanh (m'L) = [-BC_f\, Re_\delta^{1/2}\, Re_W^{3/2}\, M + [2 \frac{k_f}{k_s} Nu(1 + BC_f\, Re_\delta\, Re_W^2) - BC_f\, Re_W M^2]^{1/2}]/((2Nu (\frac{k_f}{k_s}))^{1/2} (1+BC_f\, Re_\delta\, Re_W^2)) \qquad (2.3.11)$$

Combining equation (2.3.8) with equation (2.3.5) yields

$$Re_{L,opt} = ((Re_\delta Re_W)/(2Nu \frac{k_f}{k_s}))^{1/2} \tanh^{-1}((-BC_f Re_\delta^{1/2} Re_W^{3/2} M + (2 \frac{k_f}{k_s} Nu(1 + BC_f Re_\delta Re_W^2) - BC_f Re_W M^2)^{1/2})/((2Nu \frac{k_f}{k_s})^{1/2} (1 + BC_f Re_\delta Re_W^2) \qquad (2.3.12)$$

Substituting expressions for Nu and C_f from (2.3.4) and (2.3.5), respectively, further simplifies the equation (2.3.12) and we get

$$Re_{L,opt} = \left(\frac{Re_\delta Re_W}{2Nu\ k_s/k_f} \right)^{1/2} \tanh^{-1}(y) \qquad (2.3.13)$$

where $y = \tanh(m'L)$

$$= (-1.15\ BMP\ Re_\delta^{1/2} Re_W^{3/4} + [(1+1.328\ B\ Re_\delta Re_W^{3/2}) - BM^2P^2]^{1/2})/(1 + 1.328\ Re_\delta Re_W^{3/2}) \qquad (2.3.14)$$

$$P = (k_s/k_f)^{1/2}/Pr^{1/6}$$

Similarly equation (2.3.7) combined with (2.3.4) and (2.3.5) yields

$$N_s = P/[1.15\ Re_\delta^{1/2} Re_W^{3/4} \tanh(m'L) + P\ M] + 1.328\ B\ Re_W^{1/2} Re_L \qquad (2.3.15)$$

Now, $\frac{\partial N_s}{\partial Re_W} = 0$ is solved.

Differentiating (2.3.15) with respect to Re_W and setting it equal to zero, we get

$$Re_{L,opt} = [1.5(\frac{P}{1.15})\ Re_\delta^{1/2}\ Re_W^{1/4}\ \tanh(m'L)]/[\frac{1}{2}\ \mathrm{sech}^2(m'L) + B[1.15\ Re_\delta^{1/2}\ Re_W^{3/4}\ \tanh(m'L) + P\ M]^2] \qquad (2.3.16)$$

Substitution of the value of Nu from equation (2.3.4) into the equation (2.3.13), on simplification gives

$$Re_{L,opt} = \frac{P}{1.15}\ Re_W^{1/4}\ Re_\delta^{1/2}\ \frac{1}{2}\ln(\frac{1+y}{1-y}) \qquad (2.3.17)$$

Equating (2.3.16) and (2.3.17), we get

$$\frac{P}{1.15}\ Re_W^{1/4}\ Re_\delta^{1/2}\ \frac{1}{2}\ln(\frac{1+y}{1-y})$$

$$= (1.5(\frac{P}{1.15})\ Re_\delta^{1/2}\ Re_W^{1/4}\ \tanh(m'L))/((\frac{1}{2}\ \mathrm{sech}^2(m'L)\ B\ [1.15\ Re_\delta^{1/2}\ Re_W^{3/4}\ \tanh(m'L) + P\ M]^2) \qquad (2.3.18)$$

From (2.3.14)

$$\tanh(m'L) = y$$

and $\mathrm{sech}^2(m'L) = 1 - y^2$

Therefore, equation (2.3.18) further simplifies to

$$\ln((1+y)/(1-y)) = 3\ y/(\frac{1}{2}(1-y^2) + B(1.15\ Re_\delta^{1/2} Re_W^{3/4} y + PM \qquad (2.3.19)$$

For given values of B, M, P and Re_δ, equation (2.3.19) yields the optimum value of Re_W i.e. $Re_{W,opt}$. Equation (2.3.19) has been solved using an iterative technique. An initial value for Re_W is assumed and is substituted into (2.3.14) to get y. This value of y is put into (2.3.19) to see if this equation is satisfied. The value of Re_W that satisfies this equation is $Re_{W,opt}$. Once $Re_{W,opt}$ is known, it is then substituted into (2.3.12) to obtain the value of $Re_{L,opt}$.

A special case arises when M becomes zero. The equation (2.3.17) then simplifies to

$$y = \tanh(m'L) = (v/(1+v))^{1/2} \qquad (2.3.20)$$

where, $$v = (1.328\, Re_\delta\, Re_W^{3/2})^{-1} \qquad (2.3.21)$$

Equation (2.3.19) also simplifies to

$$\ln((1+y)/(1-y)) = 3y/(0.5(1-y^2) + y^2/v) \qquad (2.3.22)$$

Combining equations (2.3.20) and (2.3.22) leads to an implicit result of the form

$$\ln((v+1)^{1/2} + v^{1/2}) = v^{1/2}(v+1)^{1/2}$$

or

$$v = \left[\frac{\ln(v^{1/2} + (v+1)^{1/2})}{(v+1)^{1/2}}\right]^2 \qquad (2.3.23)$$

Numerical solution to equation (2.3.23) is $v = 0.00905$. Using this value of v in equation (2.3.21), we get

$$Re_{W,opt} = 18.3\,(B\, Re_\delta)^{-2/3} \qquad (2.3.24)$$

Substituting the value of v in (2.3.20) and combining it with (2.3.13) and (2.3.24) yields

$$Re_{L,opt} = 0.175\ P\ B^{-1/6}\ Re_{\delta}^{1/3} \qquad (2.3.25)$$

The initial value assumed for Re_W for the solution of equation (2.3.19) was obtained from equation (2.3.24). Figs.10 and 11 summarize the numerical results for the optimum width of the fin $Re_{W,opt}$ and its optimum length $Re_{L,opt}$.

2.4 LONGITUDINAL FIN OF TRIANGULAR PROFILE:

Triangular fin has practical importance because it gives maximum heat flow per unit weight with ease of manufacture. For a longitudinal fin with arbitrary profile, the governing differential equation is [4]

$$f_1(x)\frac{d^2\theta}{dx^2} + \frac{df_1(x)}{dx}\frac{d\theta}{dx} - \frac{2h}{k_s}\theta = 0 \qquad (2.4.1)$$

where $f_1(x)$ is the cross-sectional area of the fin, θ is the temperature difference between a point on the fin surface and the surroundings, h and k_s are the surface heat transfer coefficient and the thermal conductivity of the fin material, respectively. For a unit width of the fin, the cross-sectional area function $f_1(x)$ may be replaced by a profile curve function $f_2(x)$ such that for a unit width

$$f_1(x) = 2f_2(x).$$

The general differential equation then becomes

$$f_2(x)\,\frac{d^2\theta}{dx^2} + \frac{df_2(x)}{dx}\,\frac{d\theta}{dx} - \frac{h}{k_s}\,\theta = 0 \qquad (2.4.2)$$

For a triangular fin, the profile function can be written as

$$f_2(x) = \frac{\delta_o}{2}\,(x/L) \qquad (2.4.3)$$

where δ_o is the thickness of the fin at the base and L is the length of the fin. Tip of the fin is taken as the origin. Differentiating (2.4.3) with respect to x, we get

$$\frac{df_2(x)}{dx} = \frac{\delta_o}{2L} \qquad (2.4.4)$$

Substituting this into the governing differential equation for the temperature excess θ, equation (2.4.2) yields

$$x\,\frac{d^2\theta}{dx^2} + \frac{d\theta}{dx} - m''^2 L\theta = 0 \qquad (2.4.5)$$

where $$m'' = (2h/k_s\,\delta_o)^{1/2} \qquad (2.4.6)$$

The general solution of the above equation is

$$\theta = C_1 I_o\,(2m''(xL)^{1/2}) + C_2 K_o\,(2m''(xL)^{1/2})$$

where C_1 and C_2 are the arbitrary constants and $I_o(2m''(xL)^{1/2})$ and $K_o(2m''(xL)^{1/2})$ are the modified Bessel functions of the first and second kind of the zeroth order, respectively.

In order to have a finite temperature excess Θ, at the tip of the fin where $x = 0$, the arbitrary constant C_2 must equal zero because $K_o(0)$ becomes unbounded. This leaves

$$\Theta = C_1 I_o (2m''(xL)^{1/2}) \tag{2.4.7}$$

C_1 is next evaluated using the following boundary condition at the base of the fin i.e.

$$\text{at} \quad x = L, \quad \Theta = \Theta_B \tag{2.4.8}$$

The solution of the equation (2.4.7) is

$$\Theta = \Theta_B \frac{I_o (2m''(xL)^{1/2})}{I_o (2m''L)} \tag{2.4.9}$$

Heat flow rate for a unit width of the fin is given by

$$q_o = -k_s \frac{d\Theta}{dx} \Big|_{x=L} \tag{2.4.10}$$

Evaluating the derivative of Θ at $x = L$ and substituting it into the equation (2.4.10) gives

$$q_o = 2h \Theta_B I_1 (2m''L)/(m''I_o (2m''L)) \tag{2.4.11}$$

For a width W of the fin, the heat flow rate q_B becomes

$$q_B = 2h \Theta_B W I_1(u)/(m''I_o(u)) \tag{2.4.12}$$

$$\text{where} \quad u = 2m''L \tag{2.4.13}$$

$$\Theta_B = q_B m''I_o(u)/(2h W I_1 (u)) \tag{2.4.14}$$

The minimum entropy generation analysis for a triangular fin can be done on lines similar to those adopted for the rectangular fin. However, the triangular fins involve an additional geometric parameter, namely, the half angle, α.

Assuming a laminar boundary layer flow, the following correlations have been used

$$F_D = C_f \frac{\rho U_\infty^2 LW}{\cos \alpha}; \quad C_f = 1.328 \, Re_W^{-1/2} \qquad (2.4.15)$$

and

$$h = \frac{Nu \, k_f}{W} = 0.664 \, Re_W^{1/2} \, Pr^{1/3} \frac{k_f}{W} \qquad (2.4.16)$$

Combining (2.4.6) with (2.4.16) gives

$$u = 2m''L = (2h/k_s \, \delta_o)^{1/2} \, L$$

$$= 2.30 \, (k_f/k_s)^{1/2} \, Pr^{1/6} \, Re_W^{-1/4} \, Re_{\delta o}^{-1/2} \, Re_L \qquad (2.4.17)$$

where $Re_W = \frac{U_\infty W}{\nu}$; $Re_L = \frac{U_\infty L}{\nu}$, $Re_{\delta o} = \frac{U_\infty \delta_o}{\nu}$

Substitution of (2.4.14) into equation (2.1.4) yields

$$S_{gen} = q_B^2/(2T_\infty^2 \frac{hW}{m''} \frac{I_1(u)}{I_o(u)} + q_B \, T_\infty) + F_D \, U_\infty / T_\infty \qquad (2.4.18)$$

From equations (2.4.6) and (2.4.16)

$$\frac{2hW}{m''} = 1.15(k_f \, k_s)^{1/2} \, Pr^{1/6} \, Re_W^{3/4} \, Re_{\delta o}^{1/2} \nu / U_\infty \qquad (2.4.19$$

In non-dimensional form, the entropy generation number can be expressed as

$$N_s = S_{gen}/(q_B{}^2 U_\infty/(T_\infty{}^2 \nu (k_f\ k_s)^{1/2}))$$

$$= [1.15 \frac{I_1(u)}{I_o(u)} Pr^{1/6}\ Re_W{}^{3/4}\ Re_{\delta o}{}^{1/2} + M']^{-1}$$

$$+ F_D\ T_\infty \nu (k_f\ k_s)^{1/2}/q_B{}^2 \qquad (2.4.20)$$

where $$M' = q_B\ U_\infty/(\nu T_\infty\ (k_f\ k_s)^{1/2}) \qquad (2.4.21)$$

Substituting for F_D in the above equation from equation (2.4.15), we get

$$N_s = (1.15 \frac{I_1(u)}{I_o(u)} Pr^{1/6}\ Re_W{}^{3/4}\ Re\ _o{}^{1/2} + M\ )^{-1}$$

$$+ 1.328\ B_1\ Re_W{}^{1/2}\ Re_L/\cos\alpha \qquad (2.4.22)$$

where

$$B_1 = \rho \nu^3\ T_\infty\ (k_f\ k_s)^{1/2}/q_B{}^2 \qquad (2.4.23)$$

If α is the half angle of the fin and γ, the slenderness ratio i.e.

$$\gamma = L/W \qquad (2.4.24)$$

then from the fin geometry (Fig. 4)

$$Re_L = \gamma\ Re_W \qquad (2.4.25)$$

and $$Re\ _o = 2\gamma\ Re_W \tan\alpha \qquad (2.4.26)$$

Making these substitutions into equations (2.4.17, 2.4.22), we get

$$u = 2m''L = 2.30\ (\gamma/2\tan\alpha)^{1/2}\ \ Re_W{}^{1/4}/P \qquad (2.4.27)$$

$$N_s = (1.15\,(2\gamma \tan\alpha)^{1/2}\,\frac{I_1(u)}{I_o(u)}\,Pr^{1/6}\,Re_W^{5/4} + M')^{-1} +$$

$$1.328\ B_1\ Re_W^{3/2}/\cos\alpha \qquad (2.4.28)$$

For the evaluation of the entropy generation number, the value of P has been taken as equal to 100 and that of Prandtl number as 0.696. For given values of α, B_1, M' and γ equation (2.4.28) has been numerically minimized to determine the optimum width of the fin i.e. $Re_{W,opt}$.

CHAPTER III

RESULTS AND DISCUSSIONS

Representative results of the numerical work of minimizing entropy generation rates and variations in optimum fin dimensions with respect to various parameters have been presented graphically from Fig. 5 through to Fig. 15.

Fig. 5 shows the variation in entropy generation number (N_s) with the diameter (Re_D) of the pin fin for constant B values. Entropy generation number has a clear minimum with respect to fin diameter. Reynolds number, based on the diameter of the fin, corresponding to minimum N_s is $Re_{D,opt}$. It is also observed from the curves that as B decreases, the $Re_{D,opt}$ shifts towards the right thereby increasing its magnitude. Since B is the ratio of irreversibility caused by friction to irreversibility caused by heat transfer, any decrease in its value means a higher rate of heat transfer, q_B. Higher q_B, corresponding to higher Θ_B, naturally augments the entropy generation rate. Increase in $Re_{D,opt}$ as a result of decreasing B reduces the value of Θ_B, thereby decreasing the total entropy generation rate.

Fig. 6 gives the change in $Re_{D,opt}$ with B for different values of M. As discussed earlier, with increasing B, $Re_{D,opt}$ decreases. However, the effect of M does not look significant for the range of its values considered here.

Fig. 7 presents the change in the optimum pin fin length, calculated by substituting $Re_{D,opt}$ into equation (2.2.16), with B for various M values. $Re_{L,opt}$ is also seen to decrease with increasing B. However, M does not seem to have any significant effect on $Re_{L,opt}$.

The information presented in Figs. 6 and 7 has been redrawn in Figs. 8 and 9 with a view to bring out the effect of parameter M on $Re_{D,opt}$ and $Re_{L,opt}$, respectively. These figures show clearly that variation in M does not influence much the optimum values of diameter and length of the fin for fixed values of B. However, the slight decrease observed in the values of $Re_{D,opt}$ and $Re_{L,opt}$ in the above figures towards higher values of M can be explained as follows. Since B is fixed for a particular combination of fin metal, fluid and heat flow rate, q_B, it follows that an increase in M can result only in enhancing the fluid velocity. Increased flow velocity not only increases the heat transfer rate but also the frictional irreversibility of the system. To maintain a desired heat transfer rate and to reduce the frictional irreversibility under increased flow velocity conditions, the optimum fin dimensions tend to decrease.

Fig. 10 shows the variation in optimum fin width for a rectangular fin with frictional irreversibility parameter, B, for constant M. It is found that the optimum fin width decreases as frictional irreversibility parameter, B, becomes more pronounced. Again for a given value of B, optimum width of the fin decreases with increasing M.

Fig. 11 presents the variation of optimum fin length of rectangular fin with frictional irreversibility parameter, B. Optimum fin length is seen to decrease with B. Similarly for a fixed value of B, there is a decrease in the optimum fin length with increase in M but in this case the effect of M is much more pronounced. It is to be noted that trends observed in the case of rectangular fins are in general agreement with the conclusions reached in the case of the pin fin.

Fig. 12 gives the variation of N_s with Re_W for a triangular-profile fin, for various slenderness ratios ($\gamma = 1/W$). The fin irreversibility number, N_s, assumes a minimum at a specific value of fin base width which is designated $Re_{W,opt}$.

Fig. 13 depicts the variation of $Re_{W,opt}$ for various slenderness ratios for fixed B_1, values. Regardless of the value of B_1, it is seen that the optimum width of triangular fin decreases as fin becomes more slender. Increase in γ decreases the base-stream temperature difference, θ_B, thereby decreasing the entropy generation due to heat transfer.

However, from equation (2.4.28) it is clear that entropy generation rate due to heat transfer is inversely proportional to the square root of γ whereas the frictional entropy generation rate is directly proportional to γ. The slenderness ratio, γ, therefore has a more pronounced effect on the frictional irreversibility. $Re_{W,opt}$, therefore, tends to decrease with slenderness ratio, γ, so that frictional irreversibility is reduced.

Fig. 14 exhibits the effect of M' on the optimum width of the fin. For given value of γ the optimum width is seen to decrease with increasing M'. This is also in agreement with the results obtained in the earlier two cases viz. the pin fin and the rectangular fin.

Fig. 15 delineates the variation of $Re_{W,opt}$ with B for various values of α for $M' = 1.0$ and $\gamma = 4.0$. It is apparent from the curve that the half-angle, α has a relatively small impact on the optimum fin size for minimum irreversibility. However, the optimum width of the fin decreases with increase in the half-angle, α, because of increased frictional irreversibility.

3.2 CONCLUSIONS:

Main conclusions drawn from this study are as follows

1. The optimum diameter and the length of the pin fin decrease with frictional irreversibility parameter B to

reduce the entropy generation rate due to friction.

2. Parameter M does not have any noticeable effect on the optimum geometry of the pin fin, for most of the range considered. There is, however, slight decrease in the optimum dimensions of the fin towards higher values of M.

3. The optimum dimensions of the rectangular fin decrease with frictional irreversibility parameter, B. In this case, M has a more pronounced effect on the optimum length of the fin. Both dimensions, i.e., length and width, register a decrease with M.

4. As the triangular fin becomes more slender, its optimum width decreases to reduce the effect of friction on the entropy generation number.

5. The optimum width and length of the triangular fin decrease with M'. However, this decrease is small.

6. The half angle α has a relatively minor impact on the optimum fin size for minimum irreversibility.

3.3 SUGGESTIONS FOR FUTURE WORK:

A similar study can be carried out for a heat exchanger. The heat exchanger should be optimized on component-by-component basis.

In the present study, constant fluid and fin material properties have been assumed. Thermal dependence of fin material and fluid properties e.g., k_s, ν , ρ , k_f should also be considered in the optimization analysis. Heat loss from the fin tip should be taken into account.

Thermodynamic optimization could also be studied for different types of arrays of fins e.g., regular or staggered arrangements. For this experiments will have to be performed to determine the local heat transfer and drag characteristics for the array. Also spacing between the fins and other dimensions will be determined with a view to improve the thermodynamic performance of the arrays.

Similar approach could also be adopted for other shapes of fins like trapezoidal fins, fins with parabolic profile, conical fins etc. The fin geometric parameters should be designed such that minimum irreversibility is produced.

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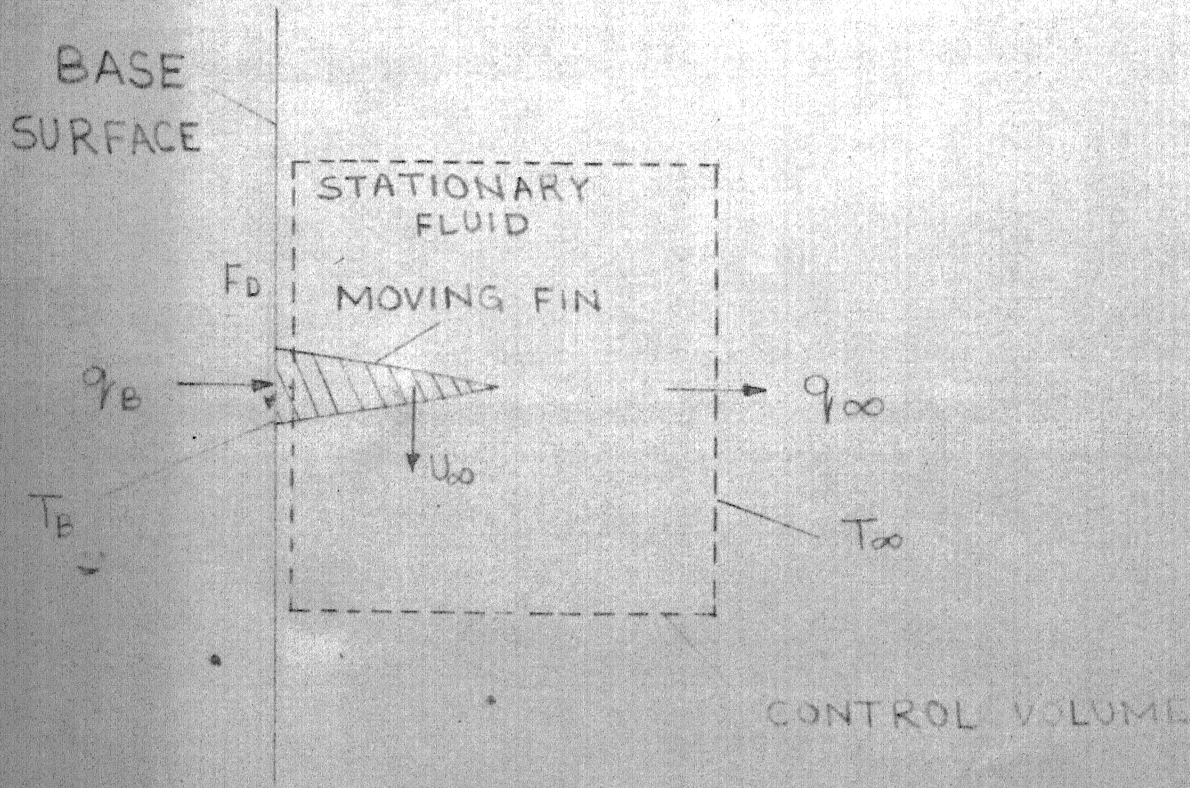

FIG. 1 SCHEMATIC DIAGRAM OF A GENERAL FIN IN A FORCED CONVECTIVE HEAT TRANSFER ARRANGEMENT.

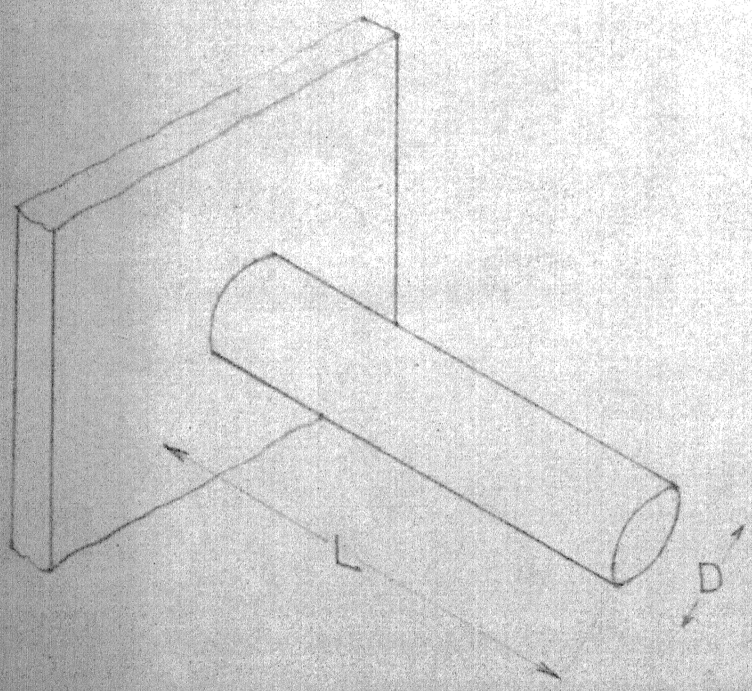

FIG. 2 PIN FIN

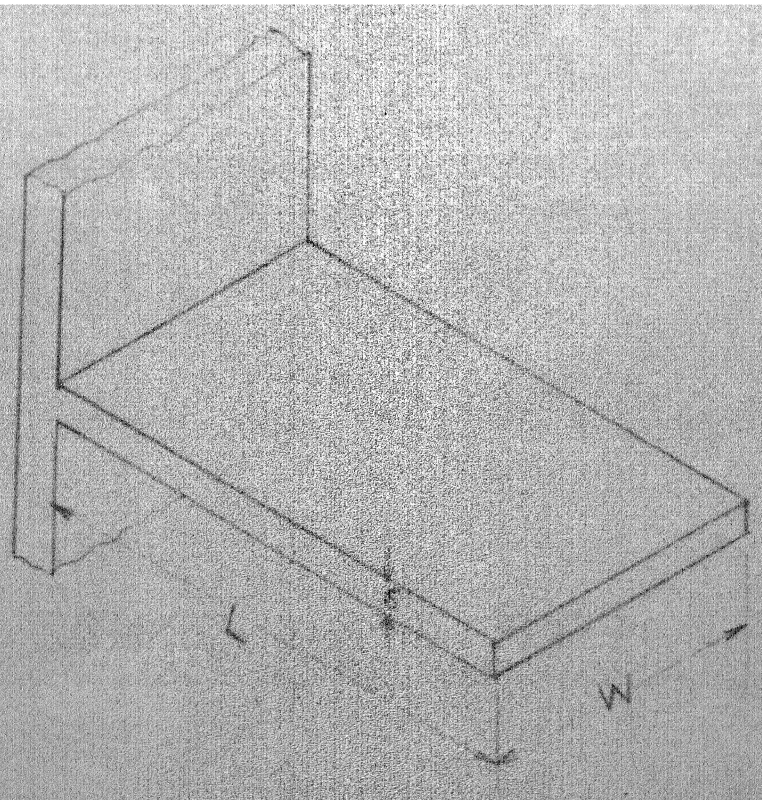

FIG. 3 RECTANGULAR FIN

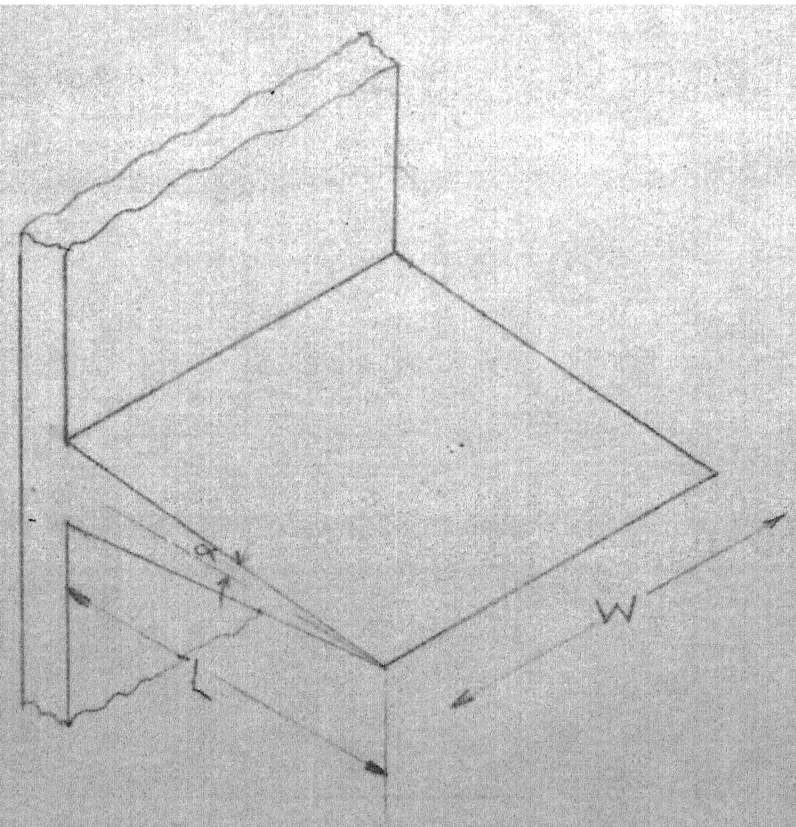

FIG. 4 LONGITUDINAL FIN WITH TRIANGULAR PROFILE

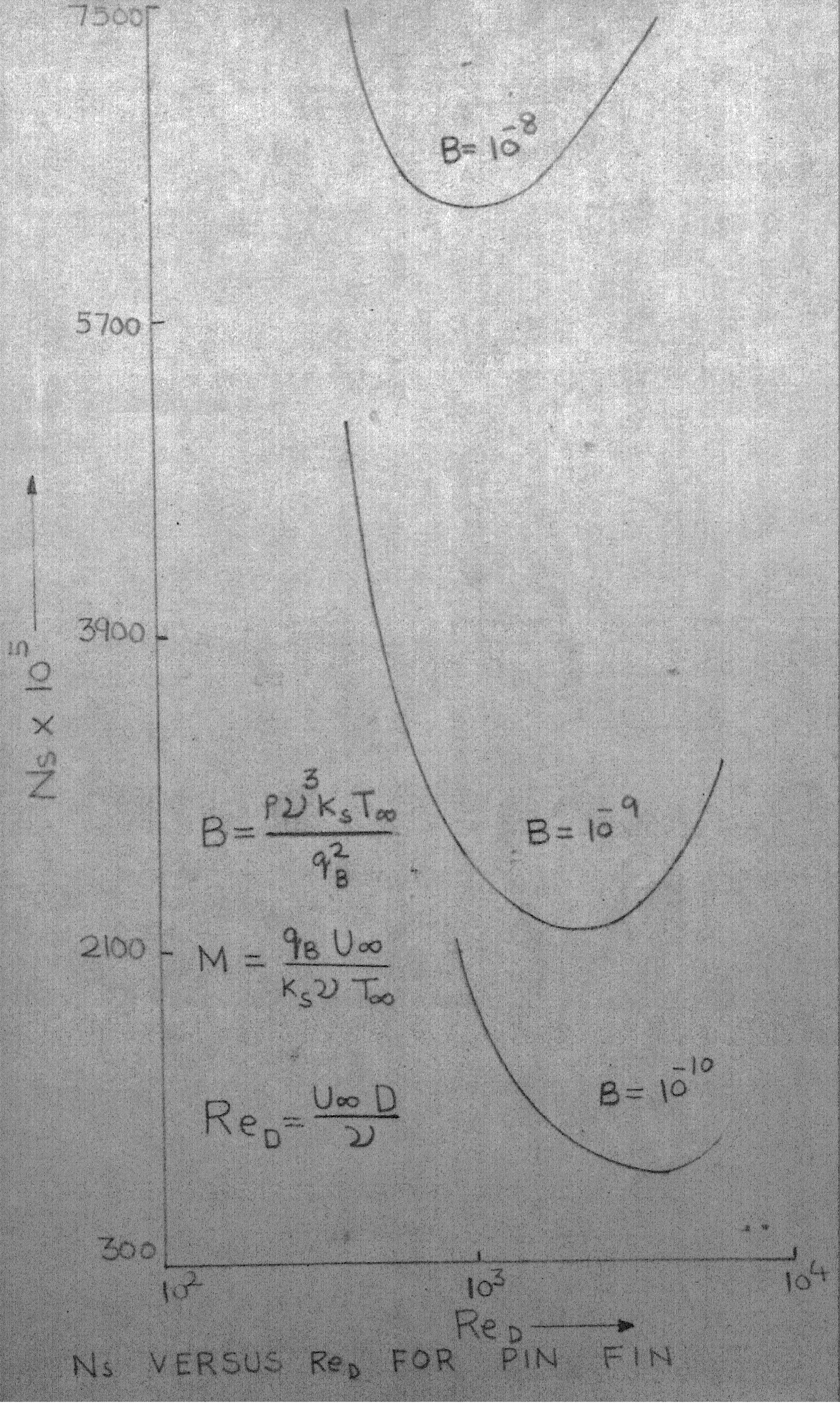

FIG. 5 N_s VERSUS Re_D FOR PIN FIN

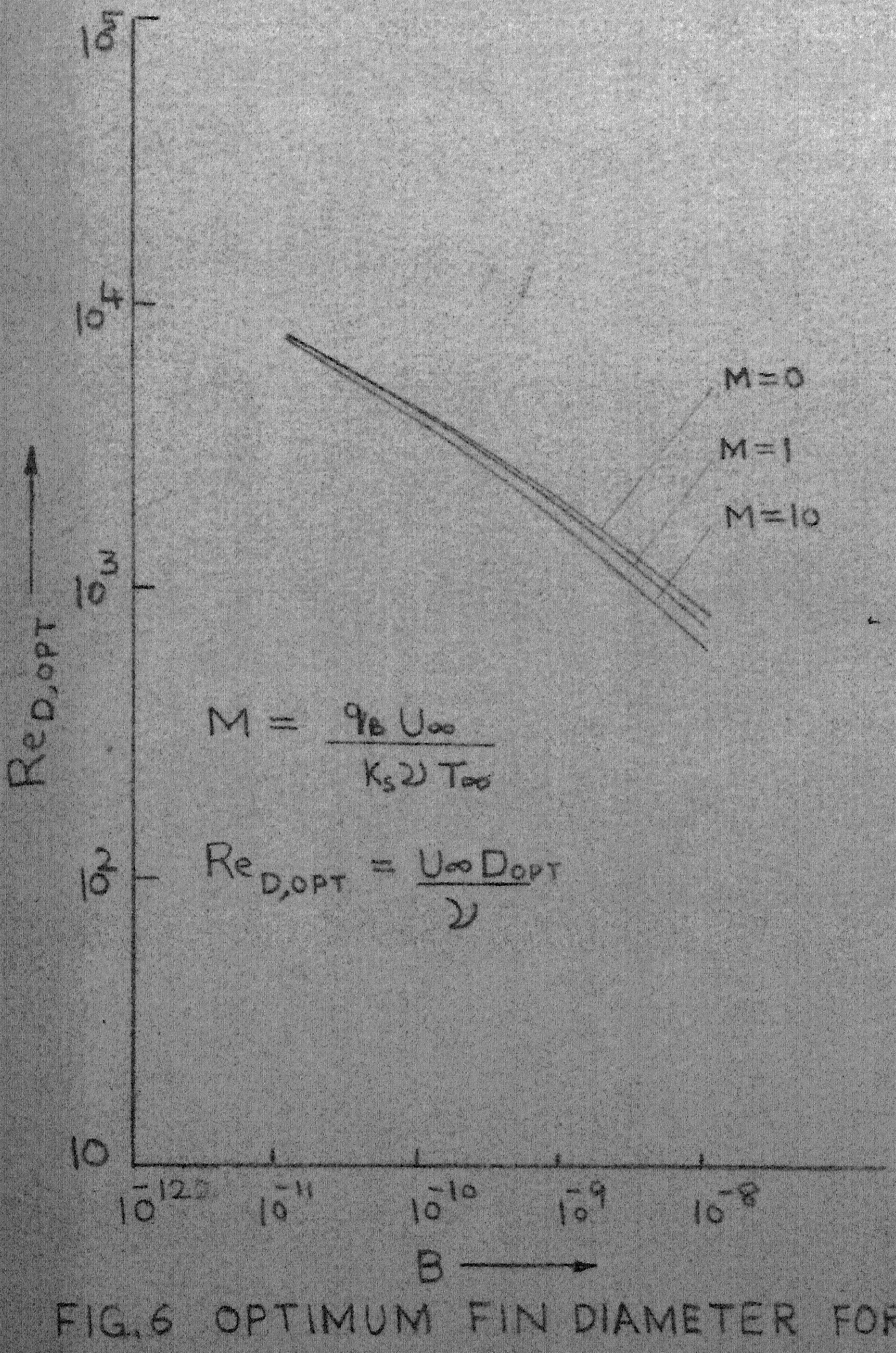

FIG. 6 OPTIMUM FIN DIAMETER FOR MINIMUM IRREVERSIBILITY

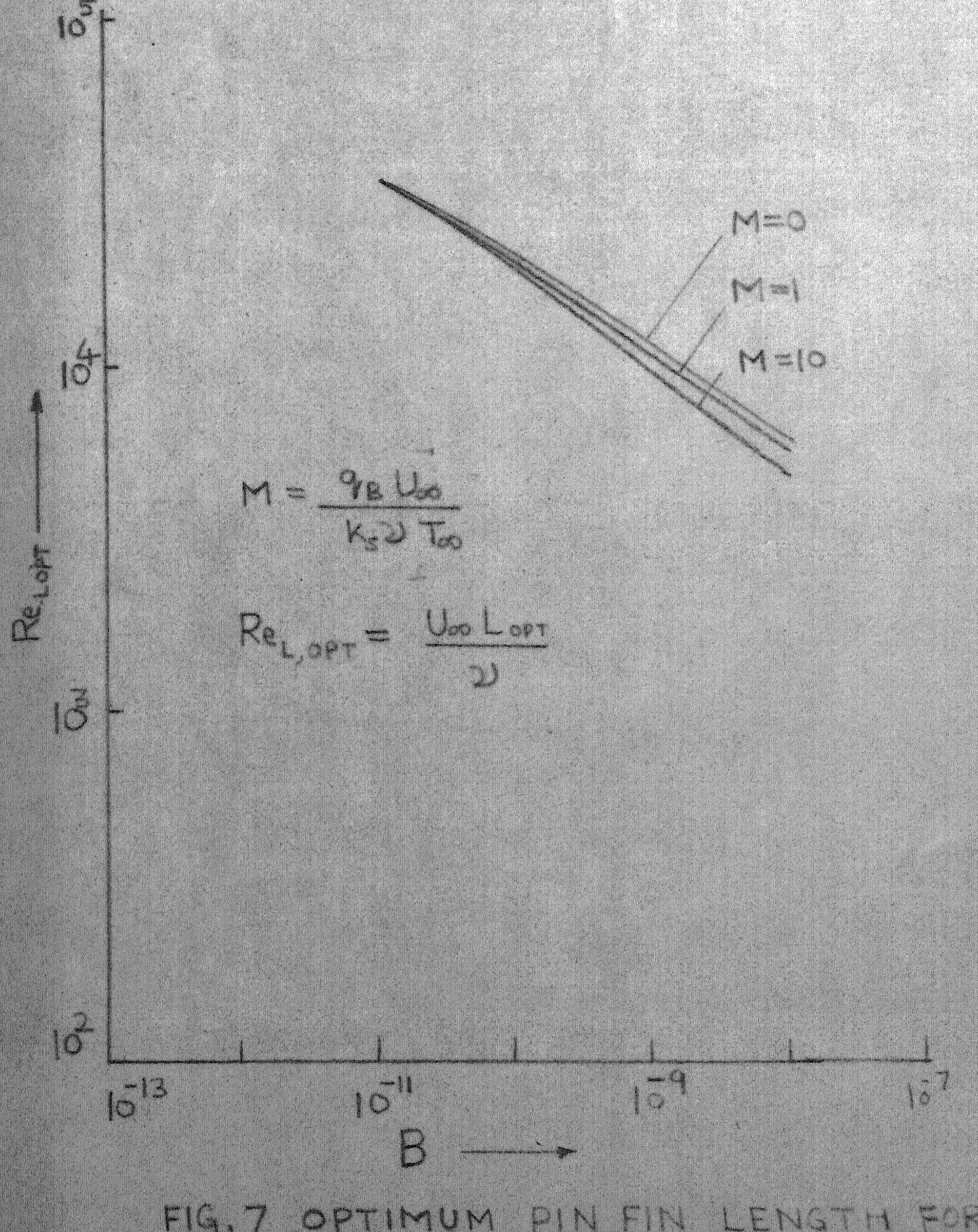

FIG. 7 OPTIMUM PIN FIN LENGTH FOR MINIMUM IRRIVERSIBILITY

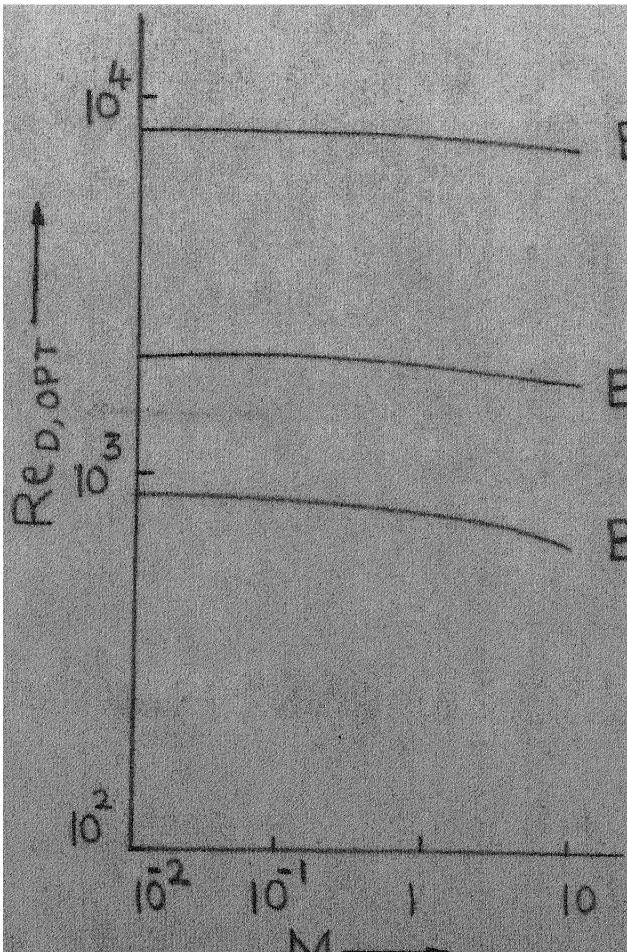

FIG. 8 OPTIMUM FIN DIAMETER VERSUS PARAMETER, M

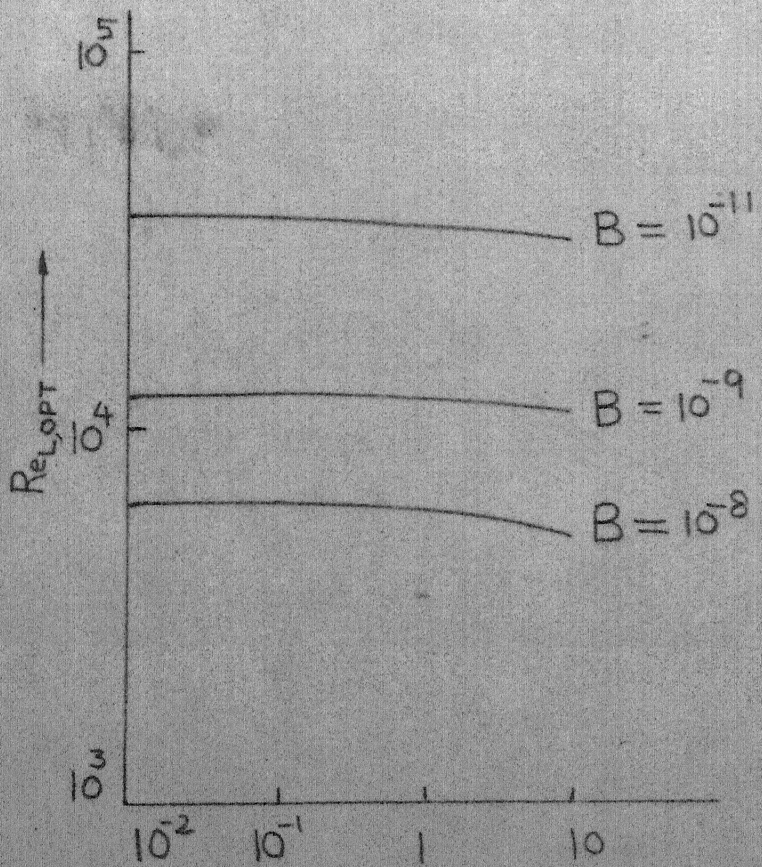

FIG. 9 OPTIMUM PIN FIN LENGTH VERSUS PARAMETER, M

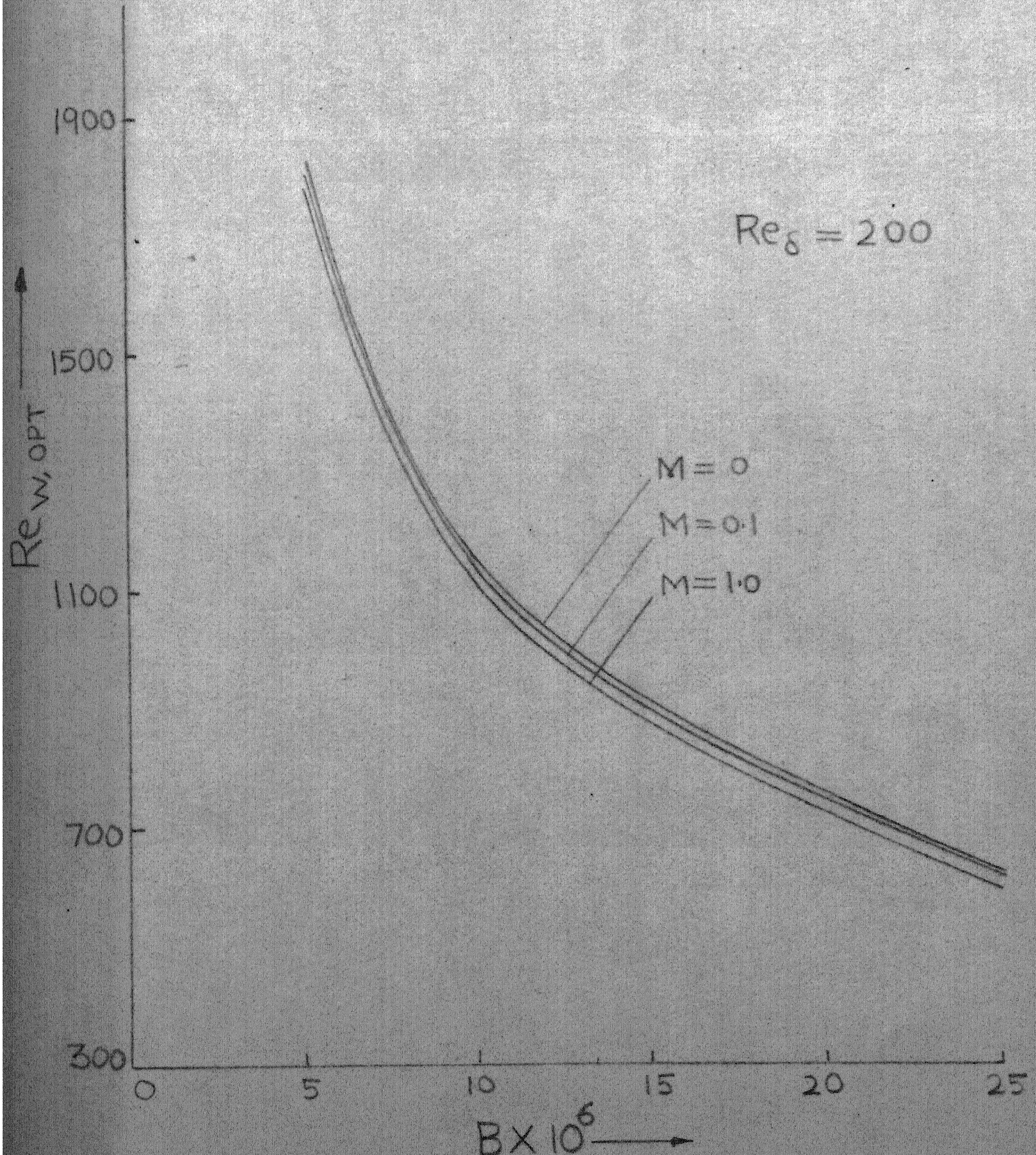

FIG. 10 EFFECT OF FRICTIONAL IRREVERSIBILITY PARAMETER, B ON THE OPTIMUM WIDTH OF THE RECTANGULAR FIN

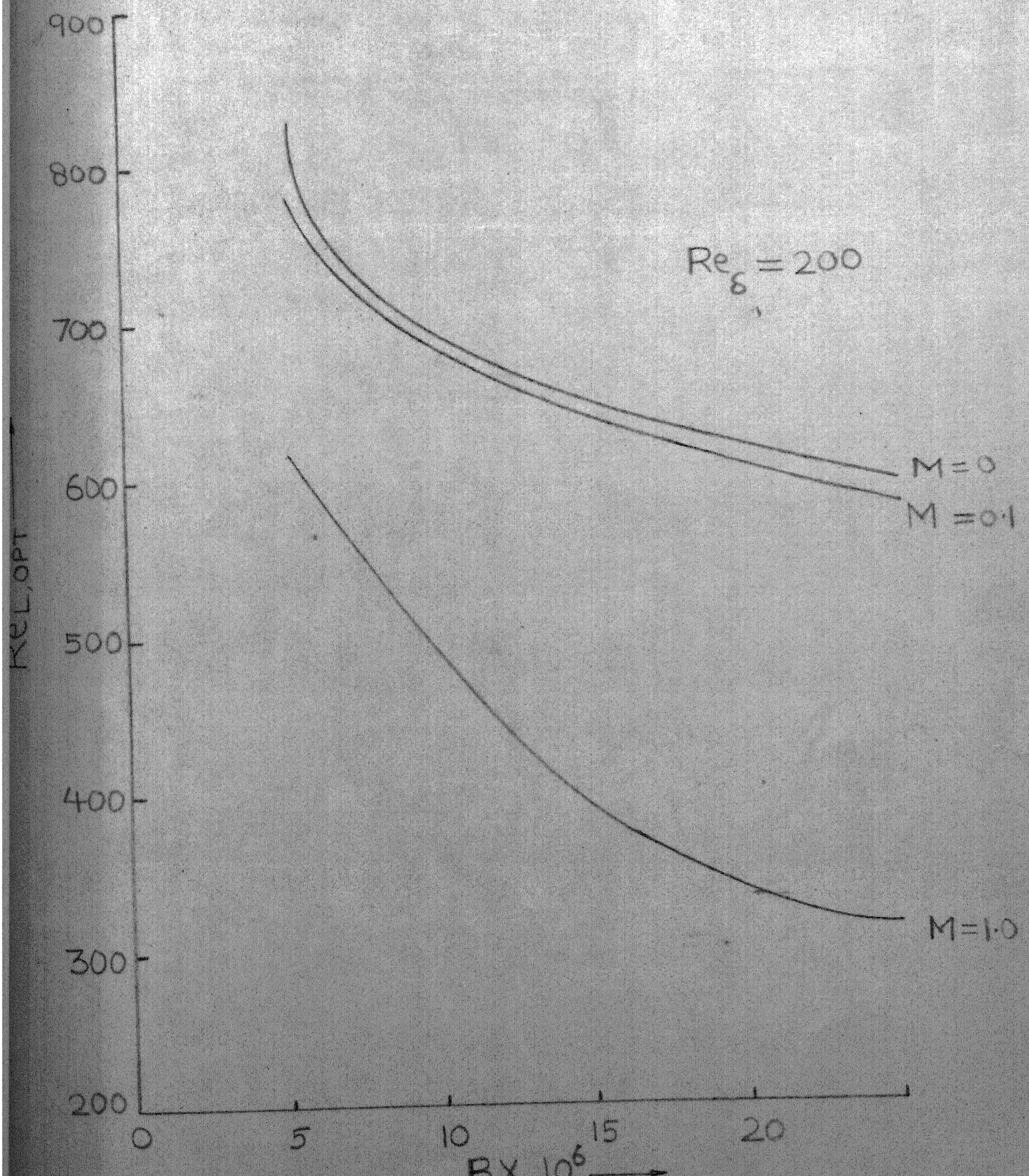

FIG. 11 EFFECT OF FRICTIONAL IRREVERSIBILITY PARAMETER, B ON THE OPTIMUM LENGTH OF THE RECTANGULAR FIN

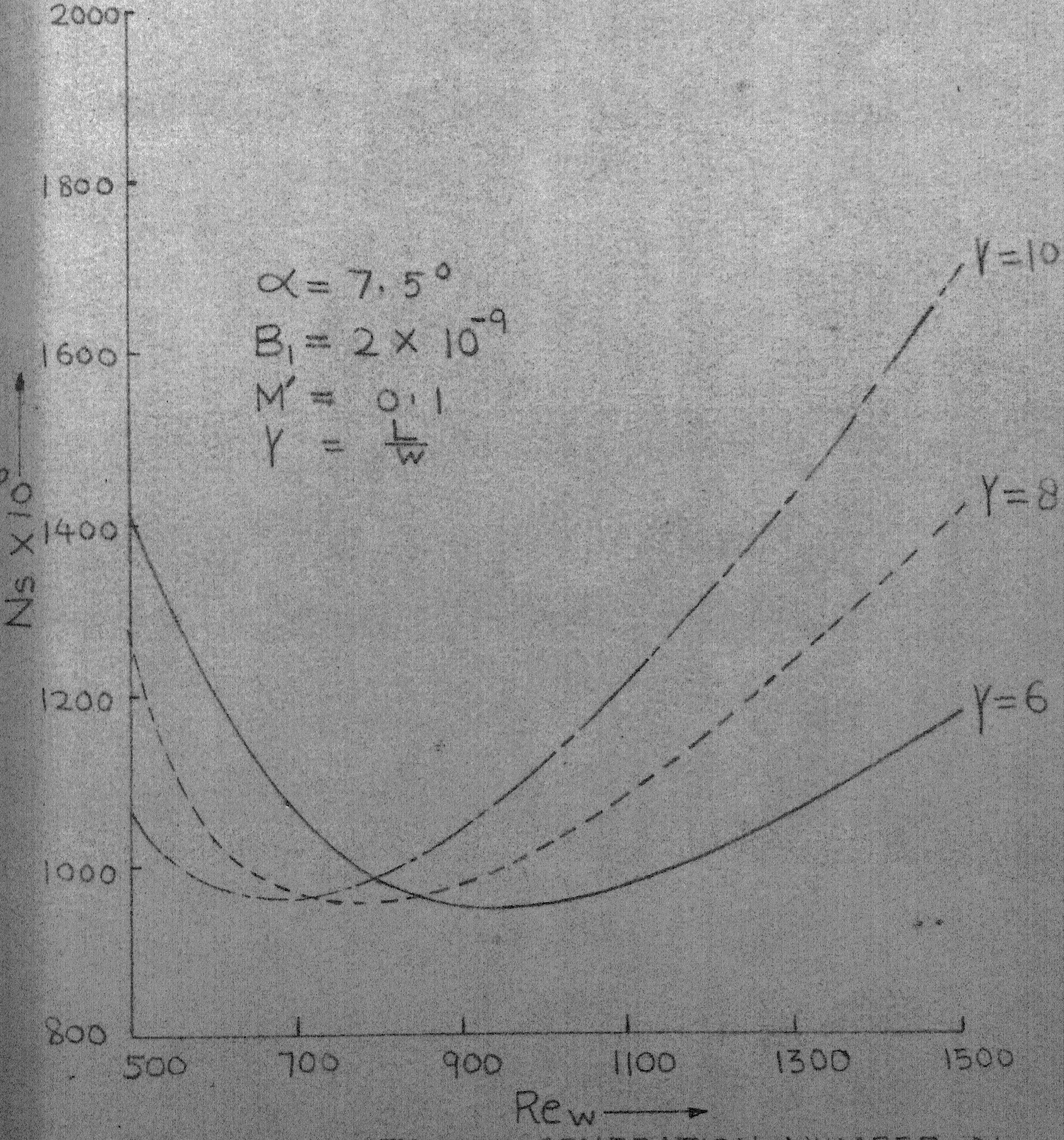

FIG. 12 ENTROPY GENERATION NUMBER Ns VS SLENDERNESS RATIO, γ FOR A TRIANGULAR-PROFILE FIN

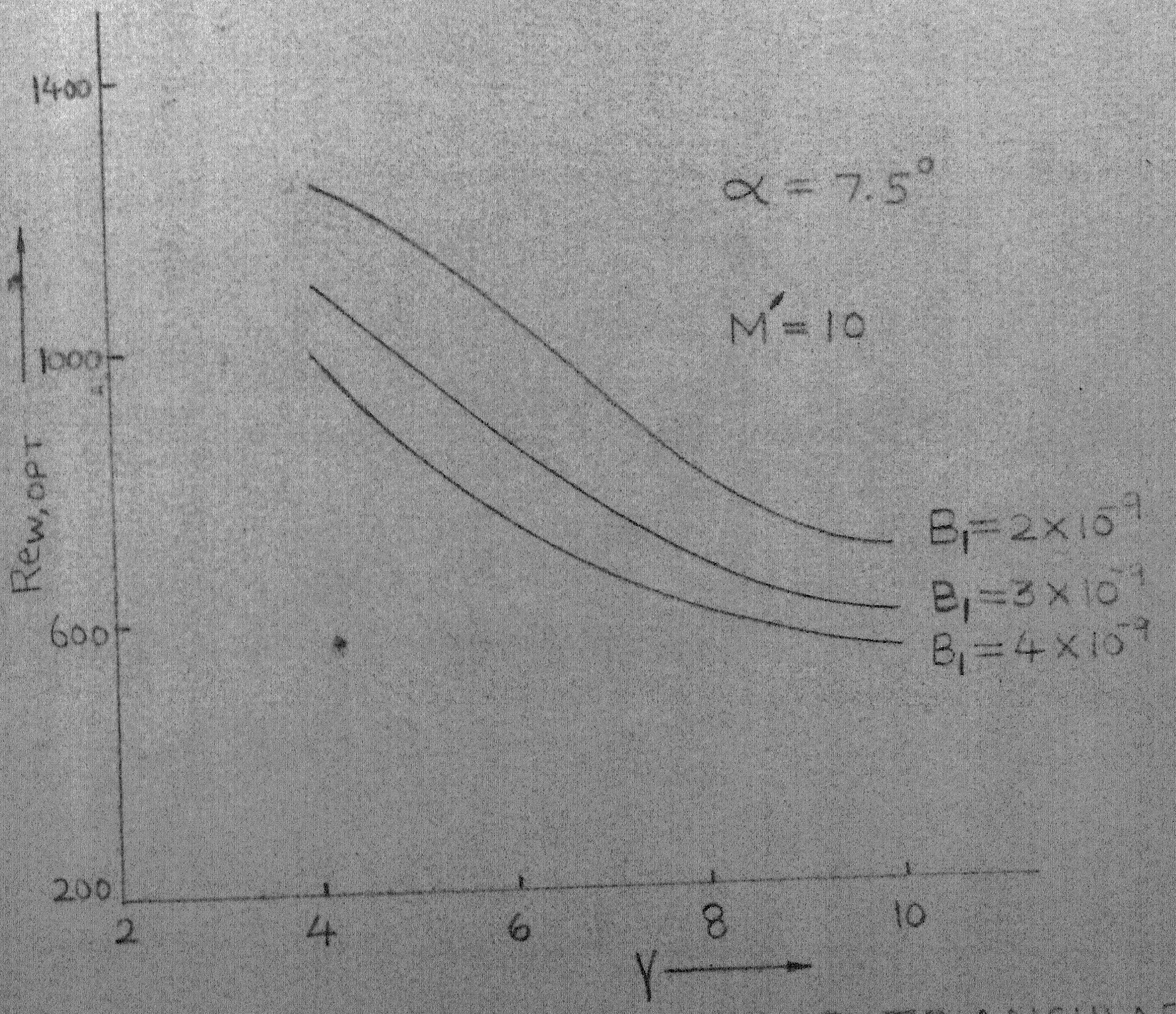

FIG. 13 OPTIMUM WIDTH OF TRIANGULAR-PROFILE FIN VS SLENDERNESS RATIO, γ

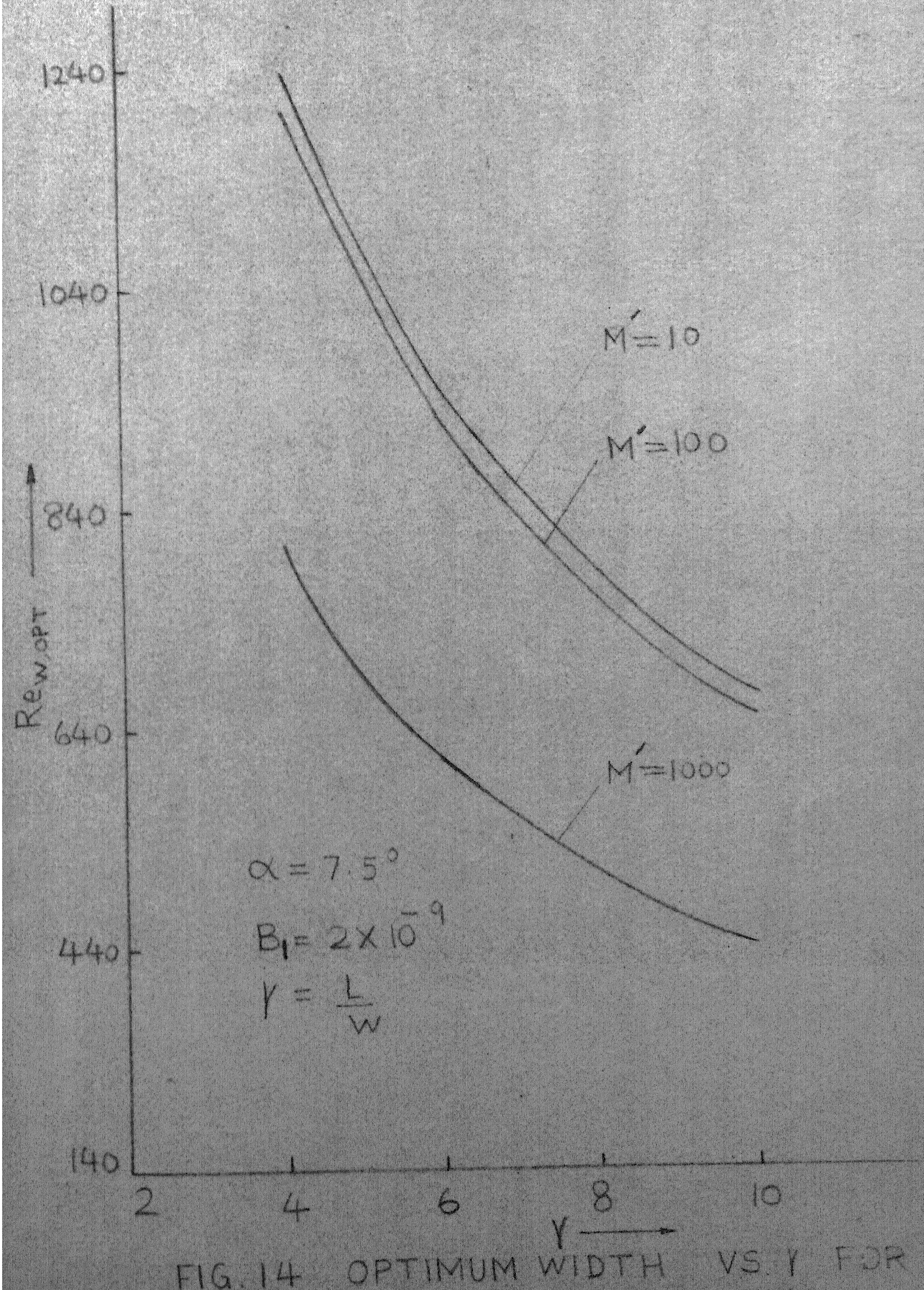

FIG. 14 OPTIMUM WIDTH VS. γ FOR TRIANGULAR-PROFILE FIN

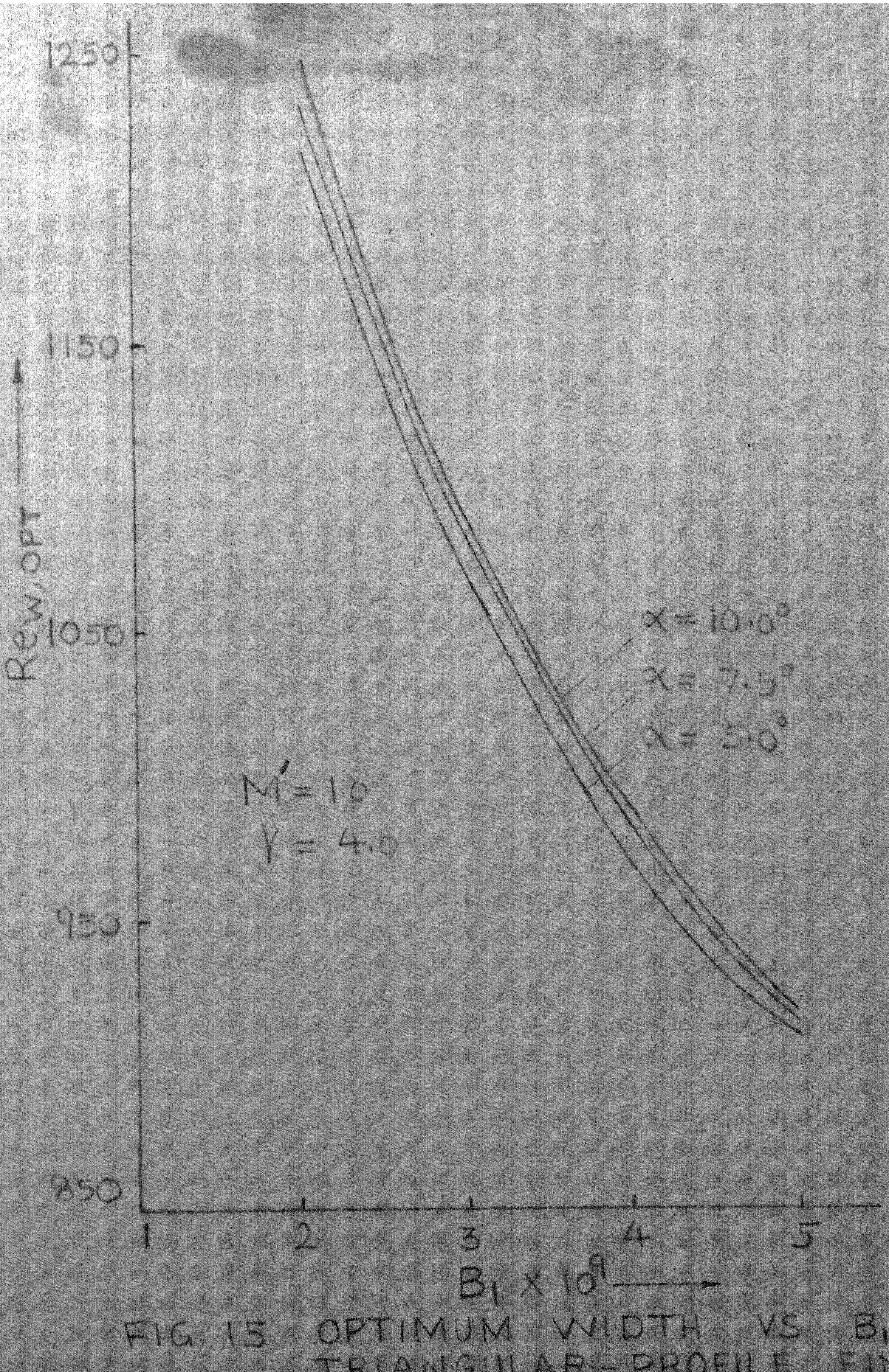

FIG. 15 OPTIMUM WIDTH VS B_1 FOR TRIANGULAR-PROFILE FIN

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